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B.Sc Part-I, Paper-II
INTEGRAL CALCULUS (Imp. Problems)

(1) Integrate $\sqrt{x^2+1}\left[\frac{\log(x^2+1)-2\log x}{x^4}\right]$

Soln.
$$I = \int \sqrt{x^2+1}\left[\frac{\log(x^2+1)-2\log x}{x^4}\right]dx$$
$$= \int \frac{\sqrt{x^2+1}}{x^4}\left[\log(x^2+1)-\log x^2\right]dx$$
$$= \int \frac{\sqrt{x^2+1}}{x^4}\cdot\log\frac{x^2+1}{x^2}\,dx$$
$$= \int \frac{1}{x^3}\cdot\sqrt{\frac{x^2+1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)dx$$
$$= \int \frac{1}{x^3}\cdot\sqrt{1+\frac{1}{x^2}}\cdot\log\left(1+\frac{1}{x^2}\right)dx$$

Put $1+\frac{1}{x^2} = t^2$

$\Rightarrow -\frac{2}{x^3}dx = 2t\,dt$

$\Rightarrow \frac{dx}{x^3} = -t\,dt$

$$I = \int \sqrt{t^2}\log(t^2)\,(-)t\,dt = -2\int t^2\log t\,dt$$
$$= -2\left[\log t\int t^2dt - \int\left(\frac{d}{dt}(\log t)\int t^2dt\right)dt\right]$$
$$= -2\left[\frac{t^3}{3}\log t - \int\frac{1}{t}\cdot\frac{t^3}{3}dt\right] = -2\left[\frac{t^3}{3}\log t - \frac{t^3}{9}\right]+c$$
$$= -\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\log\left(1+\frac{1}{x^2}\right)+\frac{2}{9}\left(1+\frac{1}{x^2}\right)^{3/2}+c$$
$$= \frac{2}{9}\left(1+\frac{1}{x^2}\right)^{3/2} - \frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\log\left(1+\frac{1}{x^2}\right)+c$$

Ans

(7) Integrate $\cos(\log x)$

Soln. $I = \int \cos(\log x)\, dx = \int x \cdot \frac{\cos(\log x)}{x} dx$

Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

and $x = e^t$

$\therefore I = \int e^t \cdot \cos t \, dt = \cos t \int e^t dt - \int \left[\frac{d}{dt}(\cos t) \int e^t dt\right] dt$

$= e^t \cos t + \int e^t \sin t \, dt$

$= e^t \cos t + \sin t \int e^t dt - \int \left\{\frac{d}{dt}(\sin t) \int e^t dt\right\} dt$

$= e^t \cos t + \sin t \cdot e^t - \int e^t \cdot \cos t \, dt$

$\Rightarrow I = e^t \cos t + e^t \sin t - I \quad [\because I = \int e^t \cos t \, dt]$

$\Rightarrow 2I = e^t(\cos t + \sin t) + c$

$\Rightarrow I = \frac{e^t}{2}(\sin t + \cos t) + c$ Ans

(8) Integrate $\frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}$

Soln. $I = \int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx = \int \frac{2x^{12} + 5x^9}{x^{15}\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} dx$

$= \int \left(\frac{2x^{12}}{x^{15}} + \frac{5x^9}{x^{15}}\right) \frac{dx}{(1 + x^{-2} + x^{-5})^3}$

$= \int \frac{2x^{-3} + 5x^{-6}}{(1 + x^{-2} + x^{-5})^3} dx$

Put $1 + x^{-2} + x^{-5} = z \Rightarrow \left(-\frac{2}{x^3} - \frac{5}{x^6}\right) dx = dz$

$\Rightarrow (2x^{-3} + 5x^{-6}) dx = -dz$

$\therefore I = \int \frac{-dz}{z^3} = (-) \frac{z^{-2}}{-2} + c = \frac{1}{2z^2} + c$

$= \frac{1}{2(1 + x^{-2} + x^{-5})^2} + c$ Ans